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ON VECTOR-POTENTIAL DISTRIBUTIONS OUTSIDE THE TOROIDAL SOLENOIDS

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A simple way of calculating the longitudinal part of the vector-potential for toroidal current systems confining the magnetic field is exemplified by the toroidal solenoid in which the current is described by the toroidal moment.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

О распределениях вектор-потенциала тороидальных соленоидов

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На примере тороидального соленоида, в котором ток описывается с помощью тороидального момента, дан простой способ вычисления продольной части вектор-потенциала для тороидальных токовых систем, конфайнмирующих магнитное поле.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

As is known /1/, the static dipole toroid moment T of a toroidal solenoid can be evaluated by a simple formula

$$T = \frac{1}{4\pi c} IV \quad \text{(in Gauss units).} \tag{1}$$

Here I is the total current on the torus surface, its volume $V = 2\pi^2\rho^2 R$, where ρ is a small and R is a large radius of the torus.

There arises a question if a charge dipole $\vec{d}$ creates a gradient electric field $\vec{E} = -(\vec{d}\vec{\nabla})\frac{\hat{\vec{r}}}{r}$; and a magnetic dipole $\vec{M}$, a magnetic field with a similar behaviour, then what field is created by a static toroidal dipole?

To answer this question let us solve the (quasi) static basic equation of electrodynamics with the gauge condition $\mathrm{div}\vec{A} = 0$ valid outside the source

$$\mathrm{rot\,rot}\,\vec{A} = c\,\vec{j}_T = c\,\mathrm{rot\,rot}\,\vec{T}\,\delta(\vec{r}), \tag{2}$$

where the right-hand side contains the "point" current of the toroidal moment $\vec{T}$. The solution of this equation is of the form of convolution of the Green function of the Laplace equation and δ-function and should be determined by the test vector functions with the appropriate analytical properties

$$\begin{aligned} <\vec{A}, \vec{K}> &= -<r^{-1}, <\mathrm{rot}'\,\mathrm{rot}'\,\vec{T}\delta(\vec{r}')\,|\,\vec{K}(\vec{r}+\vec{r}')> = \\ &= <r^{-1}, \vec{T}\,\mathrm{rot}\,\mathrm{rot}\,\vec{K}(\vec{r})> = -<\mathrm{rot}\,\mathrm{rot}\,\vec{T}\frac{1}{r}\,|\,\vec{K}(\vec{r})>. \end{aligned} \tag{3}$$

Hence, separating explicitly the contribution of a singular point r = 0 we get

$$\begin{aligned} \vec{A} &= -\mathrm{rot}(\vec{\nabla}\frac{1}{r}\times\vec{T}) + 4\pi\vec{T}\delta(\vec{r}) = -(\vec{T}\,\vec{\nabla})\frac{\hat{r}}{r} + 4\pi\vec{T}\delta(\vec{r}) = \\ &= \frac{3\vec{r}(\vec{r}\vec{T}) - r^2\vec{T}}{r^5}\Big|_{r\neq 0} + \frac{8\pi}{3}T\,\delta(r)\,. \end{aligned} \tag{4}$$

Thus, for the "point" toroid dipole $\vec{T}$ (under the condition $\mathrm{div}\vec{A} = 0$ in $R^3/\{0\}$) the distribution of the vector-potential has the same nature as $\vec{E}$ for $\vec{d}$ and $\vec{B}$ for $\vec{M}$. At the point r = 0 the local contribution of $\vec{T}$ to $\vec{A}$ is analogous to the contribution of the point magnetic dipole $\vec{M}$ of the current origin* to $\vec{B}$.

Substituting $T_z = T$ from formula (1) into (4) we get in a distant zone

$$A_z(0, 0, z) = \frac{2z^2 T_z}{4\pi z^5} = \frac{\pi I \rho^2 R}{c z^3}, \quad I = N I_0, \tag{5}$$

where I_0 is the full current in one solenoid's loop.

A more exact formula for z = 0 derived from the general expression for the vector-potential of the volume toroidal solenoid by a direct calculation [4] has the form

$$A_z(0, 0, z) = \frac{\pi I \rho^2 R}{c(z^2 + R^2)^{3/2}}. \tag{6}$$

**Difference of contributions to the field of moments of the charge and current origin has been explained in [2] (compare with [3]).*

It is seen that at the point $z = 0$ the last expression A_z appears to be automatically regularised, and at the distance of several R formula (4) is a good approximation for the case of a volume solenoid. It is seen from (4) that if $T = T(t)$, the toroidal solenoid creates the electric field.

Distributions of vector-potential for solenoids of closed configurations, more complex than the toroidal solenoid, can be estimated by expressions analogous to (4) for toroid moments of arbitrary multipolarity ℓ (we do not give them here as they are very cumbersome).

In conclusion, we should like to emphasize that by the gauge transformation the gradient part (4) can be turned to zero at any point outside the solenoid. However, its vanishing, for instance, in the half-space or strip can be achieved only with the help of singular gauge transformation (see, for example, ref. [4]). The latter is equivalent to the introduction of additional magnetic fluxes (or strings). Thus, the gradient part of (4) should be thought of as manifestation of an additional physical degree of freedom in electrodynamics. The existence of the gauge-invariant part of the vector-potential of a similar nature has been proved strictly within the Hamiltonian formalism [5].

References

1. Dubovik V.M. and Tosunyan L.A. — Fiz.Elem. Chastits At. Yadra 1983, v.14, No.5, p.1193-1228 (in Russian); J.Part.Nucl., 1989, v.14, No.5, p.504-519.
2. Ginzburg V.L. — Izv. Vuzov, Radiofizika, 1984, v.25, No.5, p.852-872 and in: Trudy FIAN, Classical and Quantum Effects in Electrodynamics v.176, M.: Nauka, p.3-31 (in Russian).
3. Khriplovich I.B. — Nesokhranenie Chetnosti v Atomnykh Yavleniyakh (Parity Nonconservation in Atomic Phenomena), M.: Nauka, (1981) first ed. p.209, sec. ed. 2 (1988), p.288.
4. Afanasiev G.N. — J.Comp.Phys., 1987, v.69, No.1, p.196;208, J.Phys.A., 1988, v.21, No.10, p.2055-2110.
5. Dubovik V.M. and Shabanov S.V. Group Representation in Physics (Tambov, Jan. 1989), In: Proceeding of Seminar, ed. V.I.Man'ko, 1989.

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